EXACT SOLUTION FOR THE EVAPORATION OF A DROP IN A SOUND FIELD WITH A STRONG TRANSVERSE FLOW OF MATTER

S. D. Traitak

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The mass transfer between a drop and a medium in the field of a sound wave was considered in [1] in the case when the amplitude of the displacement is much smaller than the radius of the drop while the wavelength is much larger. The induced steady secondary flow and diffusive transfer of volatile material in the neighborhood of the drop can then be studied in the approximation of dynamic and diffusive boundary layers. It was shown in [1] that in many practical cases the contribution from the oscillating transfer of diffusing material is small in comparison to convective transfer, and mass transfer in the neighborhood of the drop is described by the boundary-value problem

$$\frac{\partial b}{\partial \varphi}+D_2\left(\frac{\partial b}{\partial \psi}\right)_{\psi=0}\frac{\partial b}{\partial \psi}=D_2\frac{\partial^2 b}{\partial \psi^2}; \tag{1}$$

$$b|_{\psi=0}=b_1,\quad b|_{\psi\to\infty}\to b_2, \tag{2}$$

where $b=m/(m_b-1;\ m_b=m|_{\psi=0};\ \psi=u_0(x)\,r(x)\,y;\ \varphi=\int\limits_0^x u_0r^2\,dx;$

$$r(x)=R\cos(x/R);\ u_0(x)=A\sin(2x/R);\ A=1.4B^2/\omega R;\ D_2u_0r\left(\frac{\partial b}{\partial \psi}\right)_{\psi=0}=v_b;$$

m is the concentration of volatile material and m_b is its value on the surface of the drop; v_b is the Stefan flow rate on the surface of the drop; D_2 is the coefficient of diffusion; R is the radius of the drop; B and ω are the amplitude of the velocity and the frequency of the sound wave; (y, x) are local coordinates defined with respect to the surface of the drop [1].

An approximate solution of the problem (1), (2) for sufficiently small Stefan flow rate was obtained in [1]. In this case the local mass flow on the surface of the drop is given by

$$j=\rho D_2\left(\frac{\partial b}{\partial y}\right)_{y=0}=\sqrt{\frac{2}{\pi}}(b_2-b_1)\frac{\exp\left[-(b_2-b_1)^2/\pi\right]}{1+\operatorname{erf}\left[(b_2-b_1)/\sqrt{\pi}\right]}F(\bar{x}). \tag{3}$$

Here $F(\bar{x})=\rho\sqrt{\frac{AD_2}{R}}\frac{\sin 2\bar{x}\cos\bar{x}}{\sqrt{1-\cos^4\bar{x}}}$; $\bar{x}=x/R$; ρ is the density of the medium. We find an exact solution of the problem by first transforming (1) and (2) to the self-simulating variable $\xi=\psi/(2\sqrt{D_2\varphi})$. In terms of the variable ξ the boundary-value problem (1), (2) becomes

$$\frac{d^2b}{d\xi^2}+\left(2\xi-\left.\frac{db}{d\xi}\right|_{\xi=0}\right)\frac{db}{d\xi}=0; \tag{4}$$

$$b|_{\xi=0}=b_1,\quad b|_{\xi\to\infty}\to b_2. \tag{5}$$

We note that $(db/d\xi)_{\xi=0}$ is equal to a certain constant. Therefore, we can let

$$\beta=\left.\frac{db}{d\xi}\right|_{\xi=0}, \tag{6}$$

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and then the solution of (4) with conditions (5) can be written in the form

$$b = b_1 + (b_2 - b_1)\frac{\operatorname{erf}[\xi - (\beta/2)] + \operatorname{erf}(\beta/2)}{1 + \operatorname{erf}(\beta/2)}. \tag{7}$$

Using the definition of β from (6), we obtain from (7) a transcendental equation for the quantity β:

$$\beta = \frac{2}{\sqrt{\pi}}(b_2 - b_1)\frac{\exp(-\beta^2/4)}{1 + \operatorname{erf}(\beta/2)}. \tag{8}$$

The exact solution of the problem is then given by (7) and (8). The local mass flow on the surface of the drop is given by

$$j = \beta F(\bar{x}). \tag{9}$$

We note that (3) follows from (9) if we solve (8) by iteration and use the first iteration.

LITERATURE CITED

1. S. S. Kutateladze and V. E. Nakoryakov, Heat and Mass Transfer and Waves in Gas–Liquid Systems [in Russian], Nauka, Novosibirsk (1984).